Sentences and Sentence Forms

Atomic and Molecular Sentences

• From the grammatical point of view, a simple sentence comprises a subject and a predicate, either or both of which may be compound.
• To the grammarian’s stipulation, the logician adds as a necessary condition for an atomic sentence that the sentence not be divisible into more than one simple or atomic sentence.

The grammarian would consider the following to be simple sentences:

1. The tree is green.
2. The tree is not green.
3. The tree is green and tall.

The logician would consider only the first sentence to be an atomic sentence: The tree is green.

• A molecular sentence may be formed by joining two or more atomic sentences in a variety of ways, or by denying an atomic sentence.

Logicians reduce molecular sentence possibilities to five categories: conjunction, disjunction, conditional, bi-conditional, and negation.

1. A conjunction is formed by linking atomic sentences with such words as ‘and,’ ‘but,’ ‘yet.’

   Logic is easy, but thinking is hard.

Two atomic sentences are combined with the word ‘but’ to form a molecular sentence identified as a conjunction.

The following sentences differ in shades of emphasis, but the logician would consider them equivalent for purpose of analysis:

1. He was alone, and he invited a friend.
2. He invited a friend, and he was alone.
3. He invited a friend, for he was alone.
4. He was alone, but he invited a friend.
5. He was alone, so he invited a friend.
6. He was alone, yet he invited a friend.
7. He was alone; however, he invited a friend.
8. In spite of the fact that he was alone, he invited a friend.

*As the last two examples indicate, the logician is interested in sentences presented as logically coordinate, rather than in following the grammatical structure in which they are expressed.
2. A **disjunction** is formed by linking atomic sentences with ‘or.’

   He will be alone, or he will invite a friend.

Unfortunately, ‘or’ is commonly used in two different senses. One may intend ‘or’ to mean ‘one or the other, but not both.’

   Today is either Monday or Tuesday.

But one may also intend ‘or’ to mean ‘at least one, and maybe both.’

   The first baseman was either sick or tired.

In logic, ‘or’ is used to mean ‘at least one, and perhaps both.’

3. A third kind of molecular sentence is the **conditional**, usually formed by inclusion of the words ‘if . . . then.’

   If Justice cheated, [then] he will be expelled.

   • A conditional does not assert the truth of its ‘if’ clause, nor of its ‘then’ clause.
   • It claims only that, should the ‘if’ clause be true, so is the ‘then’ clause.
   • A conditional is only false when the ‘if’ clause is true, but the ‘then’ clause is false.

   Justice cheated yet was not expelled.

Whereas atomic sentences in conjunctions and disjunctions can be interchanged without changing the meaning logically, similar interchange is not possible with the conditional.

To say, ‘He is a Californian and an American.’ is logically the same as saying, ‘He is an American and a Californian.’

To say, ‘Either Elizabeth went by plane, or she took the car.’ is logically the same as saying, ‘Either Elizabeth took the car, or she went by plane.’

However, to say, ‘If Elizabeth is a Californian, she is an American.’ is not logically equivalent to saying, ‘If Elizabeth is an American, she is a Californian.’

4. The fourth type of molecular sentence is the **bi-conditional**, which inserts the words ‘if and only if’ between two atomic sentences.

   He is playing bridge if and only if he has finished his chores.

This sentence says two things:

1. If he is playing bridge, he has finished his chores.
2. If he has finished his chores, he is playing bridge.
5. The last of the five types of molecular sentences is the negation, which is the denial of an atomic sentence.

To deny an atomic sentence, ‘The moon is glowing.’ it is only necessary to insert a ‘not,’ ‘The moon is not glowing.’

It is sometimes advantageous logically to consider the above as a short form of,

‘It is not the case that the moon is glowing.’ or ‘It is false that the moon is glowing.’

Logical Notation

The system for symbolizing logical forms borrows certain basic principles from the grammar of the language of mathematics.

Logic uses symbols to show relationships between sentences.

1. Capital letters represent atomic sentences.
   Rather than write:
   
   Art is long, and life is short.
   
   we simply write:
   
   A and L

2. A dot (.) represents ‘and.’

   The dot (.) represents conjunction.
   
   The sentence:
   
   Art is long, and life is short.
   
   now becomes:
   
   A . L

3. A wedge (v) represents ‘or.’

   The wedge (v) inserted between two elements joined by ‘or’ represents disjunction.
The sentence:
   Either he will come alone, or he will invite a friend.
may be represented as:
   \( A \lor F \)

4. An \( \rightarrow \) arrow represents ‘If . . . Then.’

An arrow will represent a conditional.
The arrow is inserted between the symbols representing the condition and that representing the result.
The sentence:
   If Kermit cheated, then he will be expelled.
would now be read as:
   \( K \rightarrow E \)

5. A double-headed arrow \( \leftrightarrow \) represents ‘. . . if and only if . . .’
The bi-conditional is represented by a double-headed arrow.
Thus, the sentence:
   He is playing bridge if and only if he has finished his chores.
now becomes:
   \( B \leftrightarrow C \)

6. A tilde \( \sim \) represents ‘not’ or ‘it is not the case that . . .’

Negation is represented by a tilde \( \sim \).
Thus, the affirmative sentence:
   The sky is blue.
in its negative form,
   The sky is not blue.
becomes:
   \( \sim S \)
Defining Terms

We can now give an exact definition of the terms ‘atomic sentence’ and ‘molecular sentence.’

The special symbols introduced (., v, →, ↔, ~) are called operators.

Thus:

An atomic sentence is one which needs no operators in order to represent it in logical notation.

A molecular sentence requires at least one operator to symbolize it.

<table>
<thead>
<tr>
<th>Kind of Sentence</th>
<th>Kind of Symbol</th>
<th>Example</th>
<th>Symbol</th>
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<tr>
<td>Atomic</td>
<td>Capital Letter</td>
<td>A</td>
<td>A = The apples are ripe.</td>
</tr>
<tr>
<td>Conjunction</td>
<td>Dot</td>
<td>A . B</td>
<td>The apples are ripe and the berries are red.</td>
</tr>
<tr>
<td>Disjunction</td>
<td>Wedge</td>
<td>A v B</td>
<td>Either the apples are ripe or the berries are red.</td>
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<tr>
<td>Conditional</td>
<td>Arrow</td>
<td>A → B</td>
<td>If the apples are ripe, then the berries are red.</td>
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<td>Bi-conditional</td>
<td>Double Arrow</td>
<td>A ↔ B</td>
<td>The apples are ripe if and only if the berries are red.</td>
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<td>Negation</td>
<td>Tilde</td>
<td>~ B</td>
<td>The berries are not red.</td>
</tr>
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Exercise Examples

Translating sentences into logical notation:

a) He came on Monday, and he left on Tuesday. (M, T)

\[ M \cdot T \]

b) If it rains, the game will be postponed. (R, G)

\[ R \rightarrow G \]

c) Either she went to the theater, or she is working late.

\[ (S, W) \quad S \lor W \]
d) If he did not return to work, he must have had a reason.  \( (W, R) \)
\[
\neg W \rightarrow R
\]

Form and Meaning

Translating English sentences into logical notation often involves two stages:
1. The sentence may need revision in order to fit the pattern of one of the six kinds of sentences.
2. The revised sentence is converted into symbols.

For example, this sentence occurs in an argument by Thomas Carlyle:  That there should one man die ignorant who had capacity for knowledge, this I call a tragedy.

This sentence could be rephrased as: If a man should die ignorant, and if he had capacity for knowledge, then I would call his death a tragedy.

In logical notation:  \((I \cdot K) \rightarrow T\)

Parentheses serve as punctuation which eliminates ambiguity and suggests how the symbols are to be read.

The following interpretations of Carlyle's statement are not interchangeable:
\[(I \cdot K) \rightarrow T\]  \[I \cdot (K \rightarrow T)\]

Without the parentheses, there would be no formal indication of how the sentence should be read:  \(I \cdot K \rightarrow T\)

A condition does not need to be put in an ‘If . . .’ form in natural English. The simple sentence ‘A man died ignorant’ might appear as a conditional in any of these sentence forms:
- If a man died ignorant, . . .
- Whenever a man died ignorant, . . .
- Should a man die ignorant, . . .
- Had any man died ignorant, . . .

It is worth noting how logicians deal with the condensation which occurs in everyday language. The constituent atomic sentences are only partially stated in these examples:
1. If Hilda and Jed are coming, you don’t need to phone.

The logician will see three, not just two atomic sentences here:

2. Hilda is coming.  \((H)\)
3. Jed is coming.  \((J)\)
4. You need to phone.  \((P)\)
Properly symbolized, the sentence would read:

5. \((H \cdot J) \rightarrow \neg P\)

Example 2

1. If Chico will not make the motion, it will be up to Oroville or Red Bluff.

The three atomic sentences are:

2. Chico will make the motion. \((C)\)

3. It will be up to Oroville to make the motion. \((O)\)

4. It will be up to Red Bluff to make the motion. \((R)\)

The sentence in #1 would be properly symbolized as:

5. \(\neg C \rightarrow (O \lor R)\)

Example #3: What grammarians describe as sentences with compound predicates, logicians might develop as separate atomic sentences joined as conjunctions.

1. King Arthur was healthy, wealthy, and wise.

This sentence would become:

2. King Arthur was healthy. \((H)\)

3. King Arthur was wealthy. \((W)\)

4. King Arthur was wise. \((I)\)

This would be symbolized as:

5. \((H \cdot W \cdot I)\)